J.K.A.U.: Sci., vol. 4, pp. 145-155 (1412 A.H./1992 A.D.)

Some Remarks on Sections of a Fuzzy Matrix

F.I. SIDKY and E.G. EMAM
Dept. of Math., Faculty of Science, Zagazig University, Zagazig, Egypt

ABSTRACT. The concept of sections of a fuzzy matrix was introduced by Kim & Roush. We study the relation between a fuzzy matrix and its sections. Also, we introduce the concept of α-irreflexive, strongly irreflexive and circular fuzzy matrix.

KEYWORDS. Fuzzy matrix, Boolean matrix, section of a fuzzy matrix, circular fuzzy matrix.

1. Introduction

A Boolean matrix is a matrix with elements each has value 0 or 1. A fuzzy matrix is a matrix with elements having values in the closed interval [0,1]. The concept of sections of a fuzzy matrix was introduced by Kim and Roush[1].

In this paper, we show that many properties of a fuzzy matrix, such as reflexive, irreflexive, transitive, nilpotent, regular and others, can be extended to all its sections. We show also that some properties of the sections of a fuzzy matrix do not extend to the original fuzzy matrix, such as regularity property.

Moreover, we define some properties of a square fuzzy matrix, such as α-irreflexive, strongly irreflexive and circularity, and examine it throughout our results.

2. Preliminaries and Definitions

We shall begin with the following definitions.

Definition 2.1 [2-5]

The operations $+$, $\cdot$, $\leftarrow$ and $-$ on $[0,1]$ are defined as follows:

$$a + b = \max(a,b), \quad a \cdot b = \min(a,b),$$

$$a \leftarrow b = \begin{cases} & \text{if } a > b, \\ a & a \leqslant b, \end{cases} \qquad b \rightarrow a = a \leftarrow b,$$

$$a - b = \begin{cases} a & \text{if } a > b \\ 0 & \text{if } a \leqslant b. \end{cases}$$

where $a, b \in [0,1]$.

We shall write $a\,b$ instead of $a \cdot b$.

Remark.

A fuzzy relation R from X to Y is defined to be fuzzy subset of $X \times Y$. If X and Y are finite, we put $X = \{x_1, \ldots, x_m\}$ and $Y = \{y_1, \ldots, y_n\}$ and $R(x_i, y_j) = r_{ij}$ ($r_{ij} \in [0,1]$), $i \in I$ and $j \in J$, where $I = \{1, \ldots, m\}$ and $J = \{1, \ldots, n\}$. So, $R = [r_{ij}]$; *i.e.*, R is a fuzzy matrix. The composition of the fuzzy relations R and S on $X \times Y$ and $Y \times Z$, respectively, is defined to be a fuzzy relation $R \circ S$ on $X \times Z$ such that $R \circ S(x,z) = \operatorname{Sup}_{y \in Y} \min(R(x,y), S(y,z))$. The equation $R \circ S = T$ of fuzzy relations is called fuzzy relation equation. The problem of fuzzy relation equation is "find R knowing S and T". In order to solve this problem, Sanchez[6] introduced the operations $\leftarrow$ and $\rightarrow$. Note that the equation $R \circ S = T$ can be written in fuzzy matrix form $[r_{ij}][s_{jk}] = [t_{ik}]$, where X and Y as above and $Z = \{z_1, \ldots, z_k\}$. The product of the fuzzy matrices is defined as in the crisp case with $+$ and $\cdot$ as in the above definition.

Definition 2.2 [2-5,7]

For fuzzy matrices $A = [a_{ij}]$ $(m \times n)$, $B = [b_{ij}]$ $(m \times p)$, $D = [d_{ij}]$ $(p \times q)$, $G = [g_{ij}]$ $(m \times n)$ and $R = [r_{ij}]$ $(n \times n)$, the following operations are defined :

$$A + G = [a_{ij} + g_{ij}], \quad A \wedge G = [a_{ij}\, g_{ij}],$$

$$BD = [\sum_{k=1}^{p} b_{ik}\, d_{kj}], \quad A - G = [a_{ij} - g_{ij}].$$

$$B \leftarrow D = \prod_{k=1}^{p} (b_{ik} \leftarrow d_{kj})], \quad B \rightarrow D = \prod_{k=1}^{p} (b_{ik} \rightarrow d_{kj}$$

(where $\prod_{k=1}^{n} a_k = a_1\, a_2\, a_3 \quad a,$

$A' = [a_{ji}]$ (the transpose of A), $R^{k+1} = R^k R$ $(k = 0, 1, 2, \ldots)$,
$A / R = A - A\,R, \quad \Delta R = R - R', \quad \nabla R = R \wedge R'$,
$A \leqslant G$ if and only if $a_{ij} \leqslant g_{ij}$ for all i, j.

Definition 2.3 [3,5,8,9]

An $n \times n$ fuzzy matrix R is called reflexive if and only if $r_{ii} = 1$ for all $i = 1, 2, \ldots n$. It is called α-reflexive if and only if $r_{ii} \geqslant \alpha$ for all $i = 1, 2, \ldots n$ where $\alpha \in [0,1]$. It is called weakly reflexive if and only if if $r_{ii} \geqslant r_{ij}$ for all i, j $= 1, \ldots n$.

Definition 2.4 [2-4,7,8,10]

An $n \times n$ fuzzy matrix R is called irreflexive if and only if $r_{ii} = 0$ for all $i = 1, 2, \ldots n$.

Definition 2.5 [2,8,10]

An $n \times n$ fuzzy matrix S is called symmetric if and only if $s_{ij} = s_{ji}$ for all i, $j = 1,2,\ldots n$. It is called antisymmetric if and only if $S \wedge S' \leqslant I_n$, where I_n is the usual unit matrix.

Remark.

Note that the condition $S \wedge S' \leqslant I_n$ means that $s_{ij} \wedge s_{ji} = 0$ for all $i \neq j$ and $s_{ii} \leqslant 1$ for all i. So, if $s_{ij} = 1$, then $s_{ji} = 0$, which is the crisp case.

Lemma 2.6 [8]

Let A be an $m \times n$ fuzzy matrix. Then AA′ is weakly reflexive and symmetric.

Proof

Let $S = [s_{ij}] = AA'$. Then $s_{ii} = \sum_{k=1}^{n} a_{ik}\, a_{ik} = \sum_{k=1}^{n} a_{ik} = a_{ih}$ for some h,

$s_{ij} = \sum_{k=1}^{n} a_{ik}\, a_{jk} = a_{il}\, a_{jl}$ for some l. Therefore $s_{ij} = a_{il}\, a_{jl} \leqslant a_{il} \leqslant a_{ih} = s_i$ Hence S is weakly reflexive. Since $s_{ij} = \sum_{k=1}^{n} a_{ik}\, a_{jk}$, $s_{ji} = \sum_{k=1}^{n} a_{jk}\, a_{ik}$, $s_{ij} = s_{ji}$ and so, S is symmetric.

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Corollary 2.7

If the fuzzy matrix S is symmetric, then S^2 is weakly reflexive.

Remark 2.8

All the powers S^k; $k = 1,2,\ldots$ of a symmetric fuzzy matrix S are also symmetric and weakly reflexive.

Definition 2.9 [2-4,7,10]

An $n \times n$ fuzzy matrix N is called nilpotent if and only if $N^n = 0$ (the zero matrix).

Remark

(1) Note that, if N is an $n \times n$ fuzzy matrix with $N^m = 0$ for some positive integer m, then N is nilpotent in the sense of the above definition; *i.e.*, $N^n = 0$ (see [10]).

(2) If $N^m = 0$ and $N^{m-1} \neq 0$, $1 \leqslant m \leqslant n$, then N is called nilpotent of degree m. Note that nilpotent of degree m is nilpotent.

Definition 2.10 [2-5,7,8,10,11]

An $n \times n$ fuzzy matrix E is called idempotent if and only if $E^2 = E$. It is called transitive if and only if $E^2 \leqslant E$. It is called compact if and only if $E^2 \geqslant E$.

Remark

If E is idempotent; *i.e.*, $E^2 = E$, then we have $E^3 = E^2 = E$ and $E^4 = E^2 = E$ and so on. This means that $E^p = E$ for all $p \geqslant 2$.

Proposition 2.11

Let E be an $n \times n$ fuzzy matrix. If E is transitive and reflexive, then E is idempotent.

Proof

Since we have E is a transitive fuzzy matrix, $E^2 \leqslant E$. Now, we show that $E^2 \geqslant E$. Let $E^2 = [e_{ij}^{(2)}]$. Then $e_{ij}^{(2)} = \sum_{k=1}^{n} e_{ik}\, e_{kj} \geqslant e_{jj}\, e_{ij} = e_{ij}$ (Since we have E is reflexive).

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Proposition 2.12 [4]

Let N be an irreflexive and transitive fuzzy matrix. Then N is nilpotent.

Definition 2.13 [1,5]

An $m \times n$ fuzzy matrix A is called regular if and only if there exists an $n \times m$ fuzzy matrix G such that $AGA = A$. Such a fuzzy matrix G is called a generalized inverse or a g-inverse of A.

Remark

Note that G is not unique since it is not unique in the crisp case.

Definition 2.14 [8]

An $n \times n$ fuzzy matrix S is called similarity if and only if it is reflexive, symmetric and transitive.

3. Some Properties of Sections of Fuzzy Matrices

Definition 3.1 [1]

The section α of a fuzzy matrix A is a Boolean matrix, denoted by $A^\alpha = [a_{ij}^\alpha]$ such that $a_{ij}^\alpha = 1$ if $a_{ij} \geqslant \alpha$ and $a_{ij}^\alpha = 0$ if $a_{ij} < \alpha$.
Where $\alpha \in [0,1]$.

Lemma 3.2

For $a, b \in [0,1]$, we have the followings :

(1) $a \geqslant b \Rightarrow a^\alpha \geqslant b^\alpha$,
(2) $(a\, b)^\alpha = a^\alpha\, b^\alpha$,
(3) $(a + b)^\alpha = a^\alpha + b^\alpha$,
(4) $(a \rightarrow b)^\alpha \leqslant a^\alpha \rightarrow b^\alpha$,
(5) $(a - b)^\alpha \geqslant a^\alpha - b^\alpha$.

Proof

(1) Obvious by definition.

(2) If $a\, b \geqslant \alpha$, then $(a\, b)^\alpha = 1$, $a^\alpha\, b^\alpha = 1$. If $a\, b < \alpha$, then $(a\, b)^\alpha = 0$. Since $a\, b < \alpha$, at least one of a and b is less than α. So, $a^\alpha\, b^\alpha = 0$. Hence $(a\, b)^\alpha = a^\alpha\, b^\alpha$.

(3) If $a + b \geqslant \alpha$, then $a \geqslant \alpha$ or $b \geqslant \alpha$ or both. So, $(a + b)^\alpha = a^\alpha + b^\alpha = 1$. If $a + b < \alpha$, then $a < \alpha$ and $b < \alpha$. So, $(a + b)^\alpha = a^\alpha + b^\alpha = 0$.

(4) If $b \geqslant a$, then $(a \to b)^\alpha = a^\alpha \to b^\alpha = 1$. If $b < a$, then $(a \to b)^\alpha = b^\alpha$ and $a^\alpha \to b^\alpha = \{ {b^\alpha \atop 1}$. *So,* $(a \to b)^\alpha \leqslant a^\alpha \to b^\alpha$.

(5) If $b \geqslant a$, then $(a - b)^\alpha = 0^\alpha = \begin{cases} 0 & \text{if } \alpha > 0 \\ 1 & \text{if } \alpha = 0 \end{cases}$ and $a^\alpha - b^\alpha = 0$. If $b < a$, then $(a - b)^\alpha = a^\alpha \geqslant a^\alpha - b^\alpha$. Hence $(a - b)^\alpha \geqslant a^\alpha - b^\alpha$.

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Proposition 3.3

Let $A = [a_{ij}]\ (m \times n)$, $B = [b_{ij}]\ (m \times n)$, $R = [r_{ij}]\ (n \times n)$ and $C = [c_{ij}]\ (n \times p)$ be fuzzy matrices. Then we have the following :

(1) $A \geqslant B \Rightarrow A^\alpha \geqslant B^\alpha$,
(2) $(A \wedge B)^\alpha = A^\alpha \wedge B^\alpha$,
(3) $(A + B)^\alpha = A^\alpha + B^\alpha$,
(4) $(A \to C)^\alpha \leqslant A^\alpha \to C^\alpha$,
(5) $(A - B)^\alpha \geqslant A^\alpha - B^\alpha$,
(6) $(A\ C)^\alpha = A^\alpha\ C^\alpha$,
(7) $(A\ /\ R)^\alpha \geqslant A^\alpha\ /\ R^\alpha$,
(8) $(A')^\alpha = (A^\alpha)'$.

Proof

(1), (2), (3), (5) and (8) are clear.

(4) Let $D = A \to C$ and $F = A^\alpha \to C^\alpha$. Then

$$d_{ij} = \prod_{k=1}^{n} (a_{ik} \to c_{kj})\,)^\alpha = (a_{ih} \to c_{hj})^\alpha \quad \text{for some } h.$$

$$f_{ij} = \prod_{k=1} (a^\alpha_{ik} \to c^\alpha_{kj}) = a^\alpha_{il} \to c^\alpha_{lj} \quad \text{for some } l.$$

It follows from Lemma 3.2 that

$$f_{ij} \geqslant (a^\alpha_{ih} \to c^\alpha_{hj}) \geqslant (a_{ih} \to c_{hj})^\alpha = d_i$$

(6) Let $G = (A\ C)^\alpha$ and $P = A^\alpha\ C^\alpha$ Then

$$g_{ij} = \sum_{k=1}^{n} a_{ik}\, c_{kj})^\alpha = (a_{ih}\, c_{hj})^\alpha = a^\alpha_{ih}\, c^\alpha_{hj}, \quad \text{for some } h.$$

$$p_{ij} = \sum_{k=1}^{n} a^\alpha_{ik}\, c^\alpha_{kj} = \sum_{k=1}^{n} (a_{ik}\, c_{kj})^\alpha = (\sum_{k=1}^{n} a_{ik}\, c_{kj})^\alpha = a^\alpha_{ih}\, c^\alpha_{hi} = g_{ij}$$

(7) Let $H = (A\ /\ R)^\alpha$. It follows from Lemma 3.2, that

$$h_{ij} = (a_{ij} - \sum_{k=1}^{n} a_{ik}\, r_{kj})^\alpha \geqslant a^\alpha_{ij} - (\sum_{k=1}^{n} a_{ik}\, k_{kj})^\alpha.$$

Thus $H \geq A^{\alpha} - (A\,R)^{\alpha} = A^{\alpha} - A^{\alpha} R^{\alpha} = A^{\alpha} / R^{\alpha}$

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Proposition 3.4

Let A and B be two $m \times n$ fuzzy matrices. Then for $\alpha_1, \alpha_2 \in [0,1]$ with $\alpha_1 \leq \alpha_2$ we have :

(1) $(A + B)^{\alpha_2} \leq A^{\alpha_1} + B^{\alpha_2} \leq (A + B)^{\alpha_1}$

(2) $(A \wedge B)^{\alpha_2} \leq A^{\alpha_1} \wedge B^{\alpha_2} \leq (A \wedge B)^{\alpha_1}$

Proof

(1) $(A + B)^{\alpha_2} = A^{\alpha_2} + B^{\alpha_2} \leq A^{\alpha_1} + B^{\alpha_2} \leq A^{\alpha_1} + B^{\alpha_1} = (A + B)^{\alpha_1}$

(2) $(A \wedge B)^{\alpha_2} = A^{\alpha_2} \wedge B^{\alpha_2} \leq A^{\alpha_1} \wedge B^{\alpha_2} \leq A^{\alpha_1} \wedge B^{\alpha_1} = (A \wedge B)^{\alpha_1}$.

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Remark 3.5

The above proposition can be generalized to a finite number n of fuzzy matrices as follows :

$$\sum_{i=1}^{n} A_i)^{\max(\alpha_i;\, i = 1, \quad n)} \leq \sum_{i=1}^{n} A_i^{\alpha_i} \leq (\sum_{i=1}^{n} A_i)^{\min(\alpha_i;\, i =}$$

and

$$(\bigwedge^{n} A_i)^{\max(\alpha_i;\, i = 1, \quad n)} \leq \bigwedge_{i=1}^{n} A_i^{\alpha_i} \leq (\bigwedge_{i=1}^{n} A_i)^{\min(\alpha_i;\, i =}$$

Proposition 3.6

For an $n \times n$ fuzzy matrix A, we have

(1) $\Delta A^{\alpha} \leq (\Delta A)^{\alpha}$

(2) $\nabla A^{\alpha} = (\nabla A)^{\alpha}$

Proof

(1) $\Delta A^{\alpha} = A^{\alpha} - (A^{\alpha})' = A^{\alpha} - (A')^{\alpha} \leq (A - A')^{\alpha} = (\Delta A)^{\alpha}$

(2) $\nabla A^{\alpha} = A^{\alpha} \wedge (A^{\alpha})' = A^{\alpha} \wedge (A')^{\alpha} = (A \wedge A')^{\alpha} = (\nabla A)^{\alpha}$.

*

The following theorem is useful for decomposition of fuzzy matrices into its sections.

Theorem 3.7 [9]

Any fuzzy matrix A can be decomposed in the form :

$$A = \sum_{\alpha} \alpha A^{\alpha} \;;\; 0 < \alpha \leq 1$$

Where αA^{α} indicates that all the elements of the Boolean matrix A^{α} are multiplied by α.

Proof

Let $T = \sum_{\alpha} \alpha A$, *i.e.* $t_{ij} = \sum_{\alpha} \alpha\, a_{ij}^{\alpha}$. But $a_{ij}^{\alpha} = 0$ if $a_{ij} < \alpha$.

Then $t_{ij} = \sum_{\alpha \leqslant a_{ij}} \alpha = a_{ij}$.

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4. Relationship between a Fuzzy Matrix and Its Sections

Proposition 4.1

Let R be an $n \times n$ fuzzy matrix and $\alpha, \delta \in [0,1]$ such that $\delta \leqslant \alpha$. Then :

(1) R is α-reflexive $\Rightarrow R^{\delta}$ is reflexive,

(2) R^{α} is reflexive $\Rightarrow R$ is α-reflexive.

Proof

(1) Suppose that R is α-reflexive, *i.e.*, $r_{ii} \geqslant \alpha$. Since we have $\delta \leqslant \alpha$, $r_{ii} \geqslant \delta$ and so, $r^{\delta}_{ii} = 1$. Hence R^{δ} is reflexive for all $\delta \leqslant \alpha$.

(2) Obvious from definition of α-reflexivity.

Corollary 4.2

R is reflexive if and only if R^{δ} is reflexive for all $\delta \in [0,1]$.

Proposition 4.3

Let R be an $n \times n$ fuzzy matrix. Then R is weakly reflexive if and only if all its sections are weakly reflexive.

Proof

First, suppose that R is weakly reflexive, *i.e.*, $r_{ii} \geqslant r_{ij}$. So that $r^{\alpha}_{ii} \geqslant r^{\alpha}_{ij}$ for every $\alpha \in [0,1]$. Hence R^{α} is weakly reflexive.

Second, suppose that R^{α} is weakly reflexive for every $\alpha \in [0,1]$, *i.e.*, $r^{\alpha}_{ii} \geqslant r^{\alpha}_{ij}$. For $\alpha = r_{ij}$ we get, $r_{ii}^{r_{ij}} \geqslant r_{ij}^{r_{ij}} = 1$. Therefore $r_{ii} \geqslant r_{ij}$ and hence R is weakly reflexive.

*

Now, we define an α-irreflexive and strongly irreflexive fuzzy matrix.

Definition 4.4

An $n \times n$ fuzzy matrix R is called α-irreflexive if and only if $r_{ii} \leqslant \alpha$ for all $i = 1, 2, \ldots n$. It is called strongly irreflexive if and only if $r_{ii} \leqslant r_{ij}$ for all $i, j = 1, 2, \ldots n$.

Remark 4.5

0-irreflexive means, in fact, irreflexive.

Proposition 4.6

Let R be an $n \times n$ fuzzy matrix and $\alpha, \delta \in [0,1]$ such that $\alpha < \delta$. Then

(1) R is α-irreflexive $\Rightarrow R^{\alpha}$ is irreflexive,

(2) R^{α} is irreflexive $\Rightarrow R$ is α-irreflexive.

Proof

(1) Suppose that R is α-irreflexive. *i.e.*, $r_{ii} \leqslant \alpha$. We have $\alpha < \delta$ and so, $e_{ii} < \delta$,

i.e., R^δ is irreflexive.

(2) Obvious.

*

Corollary 4.7

Let R be an $n \times n$ fuzzy matrix. Then R is irreflexive if and only if R^δ is irreflexive for all $\delta \in [0,1]$.

Proposition 4.8

Let R be an $n \times n$ fuzzy matrix. Then R is strongly irreflexive if and only if R^α is strongly irreflexive for all $\alpha \in [0,1]$.

Proof

Suppose that R is strongly irreflexive. *i.e.*, $r_{ii} \leqslant r_{ij}$ for all $i, j = 1, 2, \dots n$. So that $r^\alpha_{ii} \leqslant r^\alpha_{ij}$. Hence R^α is strongly irreflexive.

Conversely, suppose that R^α is strongly irreflexive for all $\alpha \in [0, 1]$. Then $r^\alpha_{ii} \leqslant r^\alpha_{ij}$. Taking $\alpha = r_{ii}$ we get $r^{r_{ii}}_{ii} \leqslant r^{r_{ii}}_{ij}$, *i.e.*, $1 \leqslant r^{r_{ii}}_{ij}$. Therefore, $r_{ij} \geqslant r_{ii}$.

*

Proposition 4.9

Let S be an $n \times n$ fuzzy matrix. Then S is symmetric if and only if all its sections are symmetric.

Proof

We have S is symmetric if and only if $S = S'$ if and only if $S^\alpha = (S')^\alpha = (S^\alpha)'$.

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Proposition 4.10

A fuzzy matrix T is transitive if and only if all its sections are transitive.

Proof

We have T is transitive if and only if $T^2 \leqslant T$ if and only if $(TT)^\alpha \leqslant T^\alpha$ if and only if $(T^\alpha)^2 \leqslant T^\alpha$ if and only if T^α is transitive.

Propositions 2.11, 4.1 and 4.10 suggest that if a fuzzy matrix E is transitive and reflexive (idempotent), then all its sections are also transitive and reflexive (idempotent). This property will apply to idempotent fuzzy matrices in the following proposition.

Proposition 4.11 [11]

A fuzzy matrix E is idempotent if and only if all its sections are.

Proof

Similar to proof of proposition 4.10.

Proposition 4.12

A fuzzy matrix N is nilpotent if and only if N^α, $\alpha \in [0,1]$ is nilpotent.

Proof
Follows directly from $(N^n)^\alpha = (N^\alpha)^n$.

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Definition 4.13

An $n \times n$ fuzzy matrix C is called circular if and only if $(C^2)' \leqslant C$, or more explicitly, $c_{jk}\, c_{ki} \leqslant c_{ij}$ for every $k = 1, 2, \dots n$.

Proposition 4.14

An $n \times n$ fuzzy matrix C is circular and reflexive if and only if it is similarity.

Proof
Suppose that C is circular and reflexive. Then $c_{ij} = c_{ij}\, c_{jj} \leqslant c_{ji}$. Also, $c_{ji} = c_{ji}\, c_{ii} \leqslant c_{ij}$. So, $c_{ij} = c_{ji}$ and hence C is symmetric.

Also, we have $c_{ij}^{(2)} \leqslant c_{ji} = c_{ij}$, *i.e.*, C is transitive. Hence C is similarity.

Conversely, suppose that C is similarity. Then $c_{ij}^{(2)} \leqslant c_{ij} = c_{ji}$. Hence C is circular.

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Proposition 4.15

An $n \times n$ fuzzy matrix C is circular if and only if all its sections are.

Proof
We have C is circular if and only if $(C^2)' \leqslant C$ if and only if $((C^2)')^\alpha \leqslant C^\alpha$ if and only if $((C^2)^\alpha)' \leqslant C^\alpha$ if and only if $((C^\alpha)^2)' \leqslant C^\alpha$.

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Proposition 4.16

Let C be an $n \times n$ fuzzy matrix. Then C is compact if and only if all its sections are.

Proof
We have C is compact if and only if $C^2 \geqslant C$ if and only if $(C^2)^\alpha \geqslant C^\alpha$ if and only if $(C^\alpha)^2 \geqslant C^\alpha$.

*

Proposition 4.17

Let A be a regular fuzzy matrix with a g-inverse G, then A^α is regular with a g-inverse G^α for every $\alpha \in [0,1]$.

Proof
Since A is regular with g-inverse G, we have $A = A\,G\,A$.
Then $A^\alpha = (A\,G\,A)^\alpha = A^\alpha\, G^\alpha\, A^\alpha$. Hence A^α is regular and G^α is a g-inverxe of it.

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The following example shows that the converse of the above proposition is not true in general.

Example 4.18

We consider the fuzzy matrix.

$$A = \begin{matrix} 0.7 & 0.3 & 0 \\ 0.3 & 0.7 & 0.3 \\ 0.3 & 0.3 & 0.7 \end{matrix}$$

Let the two sections

$A^{0.3} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ and $A^{0.7} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ of A be regular with g-inverses

$G = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, respectively. Since $A^{0.3} > A^{0.}$

we have $G > I$; *i.e.*, G is reflexive

So, $A^{0.3}\, G\, A^{0.3} = \begin{matrix} 1 & 1 & \\ & & \\ & 1 & \end{matrix}$, wich contradicts the regularity of $A^{0.3}$.

Hence A is not regular.

References

[1] **Kim, K.H.** and **Roush, F.W.**, Generalized fuzzy matrices, *Fuzzy Sets and Systems* **4:** 293-315 (1980).

[2] **Hashimoto, H.**, Canonical form of a transitive fuzzy matrix, *Fuzzy Sets and Systems* **11:** 157-162 (1983).

[3] **Hashimoto, H.**, Decomposition of Fuzzy Matrices, *Siam J. Alg. Disc. Math.* **6:** 32-38 (1985).

[4] **Hashimoto, H.**, Reduction of a Fuzzy Retrieval Model, *Information Sciences* **27:** 133-140 (1982).

[5] **Hashimoto, H.**, Subinverses of fuzzy matrices, *Fuzzy Sets and Systems* **12:** 155-168 (1984).

[6] **Sanchez, E.**, Resolution of composite fuzzy relation equations, *Inform and Control* **30:** 38-48 (1976).

[7] **Hashimoto, H.**, Reduction of a Nilpotent Fuzzy Matrix, *Information Sciences* **27:** 233-243 (1982).

[8] **Dubois, D.** and **Prade, H.**, *Fuzzy sets and Systems, Theory and Application,* Academic Press, New York (1980).

[9] **Kaufmann, A.**, *Introduction to the Theory of Fuzzy Subjects,* Academic Press, New York (1975).

[10] **Hashimoto, H.**, Szpilarajn's theorem on fuzzy orderings, *Fuzzy Sets and Systems* **10:** 101-108 (1983).

[11] **Hashimoto, H.**, Convergence of powers of a fuzzy transitive matrix, *Fuzzy Sets and Systems* **9:** 153-160 (1983).

بعض الملحوظات على مقاطع المصفوفة الفازية

ف . صدقي و إ . ج . إمام
قسم الرياضيات ، كلية العلوم ، جامعة الزقازيق ، الزقازيق ، مصر

المستخلص . فكرة مقاطع المصفوفة الفازية استنتجها كيم وروش عام ١٩٨٠م . ويتناول هذا البحث دراسة العلاقة بين المصفوفة الفازية ومقاطعها ، كما نُعرف الأفكار التالية للمصفوفة الفازية :

α-irreflexive, strongly irreflexive and circular fuzzy matrix.

نعطي في القسم الأول من هذا المقال ، مقدمة نُعرِّف فيها المصفوفة الفازية والفرق بينها وبين المصفوفة البولينية Boolean matrix ، ونشير إلى ما سوف ندرسه في هذا البحث .

وفي القسم الثاني ، نذكر بعض التعاريف والنظريات الأساسية الموجودة في المراجع والتي سوف نستخدمها خلال هذا المقال .

في القسم الثالث ، نقدم العديد من خصائص المصفوفات الفازية (مع البرهان) ومدى انتقال هذه الخصائص إلى المقاطع .

في القسم الرابع ، نبرهن العديد من العلاقات بين المصفوفة ومقاطعها من حيث الأفكار المختلفة المعرفة للمصفوفة الفازية ، مثل الانعكاس والتماثل والنقل وضد الانعكاس وضد التماثل .